I Semester B.C.A. Degree Examination, November/December 2016  
(CBCS) (F+R)  
(2014-15 & Onwards)  
BCA – 105 : DISCRETE MATHEMATICS

Time : 3 Hours  Max. Marks : 100

**Instruction :** Answer all Sections.

**SECTION – A**

1. **Answer any ten:**  
   \((10 \times 2 = 20)\)

1) If \(A = \{x | x \in N \text{ and } x < 3\}\) and \(B = \{0, 1, 3\}\). Find \(A - B\).

2) If \(A = \{1, 2, 3\}\), \(B = \{3, 4, 5\}\) and \(C = \{0, 2, 3\}\), find \((A \cap B) \times C\).

3) Construct truth table for the proposition \(p \lor \sim q\).

4) Find \(x, y, z\) if \[
\begin{bmatrix}
4 - y & 3 \\
x & 5
\end{bmatrix}
\begin{bmatrix}
-1 & z + 1 \\
1 & 5
\end{bmatrix}.
\]

5) If \(A = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}\) and \(B = \begin{bmatrix} 2 & 0 & 3 \\ 3 & 1 & 4 \end{bmatrix}\), find \(AB\).

6) Find the characteristic equation of the matrix \[
\begin{bmatrix}
1 & -2 \\
3 & 0
\end{bmatrix}.
\]

7) Prove that \(\log_a a \cdot \log_b b \cdot \log_c c = 1\).

8) Find \(n\) if \(2 \times P_2 = n P_5\).

9) On the set of integers \(Z\), the binary operation \(\cdot\) is defined by \[
a \cdot b = \frac{ab}{3}, \ \forall a, b \in Z\]. Find identity element.

10) If \(\vec{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}, \vec{b} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}\) find unit vector along \(\vec{a} - \vec{b}\).

P.T.O.
11) Find the midpoint of line joining (−2, 8) and (1, −2).

12) Find the equation of the line passing through (−1, 2) and having slope 3.

II. Answer any six of the following: (6×5 = 30)

13) If \( A = \{1, 4\}, B = \{2, 3, 6\}, C = \{2, 3, 7\} \) then verify that \( A \times (B + C) = (A \times B) + (A \times C) \).

14) Show that the function \( f : \mathbb{R} \rightarrow \mathbb{R} \) defined by \( f(x) = 4x + 3 \) is invertible. Find the inverse of \( f \).

15) Show that \( p \lor (q \land r) \leftrightarrow [(p \lor q) \land (p \lor r)] \) is a tautology.

16) If \( (p \rightarrow q) \land (p \land r) \) is given to be false, find the truth values of \( p, q, r \).

17) Write the truth table of \( (p \lor q) \lor \neg p \). Show that the compound propositions \( p \land q \) and \( \neg (p \rightarrow q) \) are logically equivalent.

18) Find the inverse of the matrix \( A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & 3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \).

19) Using Cramer’s rule solve \( 3x - y + 2z = 13; 2x + y - z = 3; x + 3y - 5z = -8 \).

20) Verify Cayley Hamilton theorem for the matrix \( \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \).

III. Answer any six of the following: (6×5 = 30)

21) If \( \log \left( \frac{a - b}{5} \right) = \frac{1}{2} \left( \log a + \log b \right) \), show that \( a^2 + b^2 = 27ab \).

22) Find the number of three digit even numbers that can be formed using 2, 3, 4, 5, 6 repetitions being not allowed.

23) If \( \binom{n+2}{2} : \binom{n-2}{4} = 57 : 16 \) find \( n \).
24) Prove that the set \( G = \{ 3n \mid n \in \mathbb{Z} \} \) is an abelian group w.r.t. addition.

25) Prove that the set \( G = \{ 2, 4, 6, 8 \} \) is an abelian group w.r.t. multiplication modulo 10.

26) If \( \mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} \), \( \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \) find \( (\mathbf{a} + 2\mathbf{b} \cdot (2\mathbf{a} - \mathbf{b}) \).

27) Show that the points \( A(1,2,3) \), \( B(2, 3, 1) \) and \( C(3,1,2) \) are vertices of an equilateral triangle.

28) If the vectors \( 4\mathbf{i} + 11\mathbf{j} + m\mathbf{k} \), \( 7\mathbf{i} + 2\mathbf{j} + 6\mathbf{k} \) and \( \mathbf{i} + 5\mathbf{j} + 4\mathbf{k} \) are coplanar, then find 'm'.

SECTION - D

IV. Answer any four of the following. (4x5 = 20)

29) Prove that the points \( (6, 4), (7, -2), (5, 1), (4, 7) \) form vertices of a parallelogram.

30) The three vertices of a parallelogram taken in order are \( (8,5), (-7, -5) \) and \( (-5, 5) \). Find the co-ordinate of the fourth vertex.

31) Find the equation of the locus of a point which moves such that its distance from X-axis is twice its distance from Y-axis.

32) Derive the equation of the straight line whose x-intercept is 'a' and y-intercept is 'b'.

33) Find 'K' for which the lines \( 2x - ky + 1 = 0 \) and \( x + (k+1)y - 1 = 0 \) are perpendicular.

34) Find the equation of straight line which is passing through intersection of the lines \( 2x - 3y - 4 = 0 \) and \( 2x + 2y - 1 = 0 \) and perpendicular to the line \( x + 4y - 8 = 0 \).
I Semester B.C.A. Degree Examination, November/December 2015  
(CBCS) (Y2K14 Scheme)  
BCA – 105 T: DISCRETE MATHEMATICS

Time: 3 Hours  
Max. Marks: 100

**Instruction**: Answer all Sections.

**SECTION – A**

I. Answer any ten:  
(10×2=20)

1) If \( A = \{2, 3, 4, 8\}, B = \{1, 3, 4\} \) and \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \).  
Verify \( A \cap B = A \cap B \).

2) If \( A = \{2, 3, 5\}, B = \{4, 5, 6\} \) and \( C = \{1, 2\} \) find \( A \times B \).

3) Define Tautology.

4) Define diagonal matrix.

5) If \( 2Y + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} \), find \( Y \).

6) State Cayley Hamilton theorem.

7) If \( \log_7 x + \log_7 x^2 + \log_7 x^3 = 6 \), find \( x \).

8) Define combination.

9) Define Abelian group.

10) If \( \vec{a} = 2i + 3j - 4k \), \( \vec{b} = 3i - 4j - 5k \) find \( |\vec{a} + \vec{b}| \).

11) Find the distance between the point, \( A = (-7, 4) \) and \( B = (-5, -1) \).

12) Find the equation of the line with slope 2 and cutting off an intercept 3 on \( Y\)-axis.
SECTION - B

II. Answer any six of the following: \((6 \times 5 = 30)\)

13) If \(A = \{a, b, c, d\}, B = \{c, d\}\) and \(C = \{d, e\}\) find \(A - B, (A - B) \cap (B - C), B \times C\).

14) If \(f : \mathbb{R} \to \mathbb{R}\) is defined by \(f(x) = 2x + 5\), prove that \(f\) is one-one and onto.

15) Prove that \((p \land q) \land (p \lor q)\) is a contradiction.

16) Write the converse, inverse and contra positive of
   "If I work hard then I get a grade".

17) Find the truth values of the propositions \(p, q\) and \(r\), if the compound proposition \((p \to \sim q) \to r\) is false.

18) If \(2A + B = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}, A - 2B = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}\) then find \(A\) and \(B\).

19) If \(A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}\), find \(A^{-2}\) using Cayley Hamilton theorem.

20) Solve the equations \(5x + 2y = 4, 7x + 3y = 5\) using Matrix method.

SECTION - C

III. Answer any six of the following: \((6 \times 5 = 30)\)

21) If \(\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)\), show that \(a = b\).

22) In how many ways the letters of the word “EVALUATE” be arranged so that all vowels are together.

23) If \(^{15}C_{r+3} = ^{15}C_{2r-3}\), find \(r\).

24) If \(G = \{3^n : n \in \mathbb{Z}\}\), prove that \(G\) is an abelian group under multiplication.

25) Prove that \(G = \{1, 5, 7, 11\}\) is a group under multiplication modulo 12.

26) Find the value of \(\lambda\) for which the vectors \(\vec{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}\) and \(\vec{b} = \mathbf{i} + \lambda\mathbf{j} - 3\mathbf{k}\) are perpendicular to each other.

27) Find the area of the triangle whose vertices are \(A(1, 2, 3), B(2, 5, 1)\) and \(C(-1, 1, 2)\) using vector method.

28) If the vectors \(2\mathbf{i} - 3\mathbf{j} + mk, 2\mathbf{i} + \mathbf{j} - k\) and \(6\mathbf{i} - \mathbf{j} + 2\mathbf{k}\) are coplanar, find \(m\).
IV. Answer any four of the following: (4x5=20)

29) Show that the points (3, 2), (0, 5), (−3, 2) and (0, −1) are the vertices of a square.

30) Find the ratio in which the x-axis divides the line segment joining the points (7, −3) and (5, 2).

31) Find the equation of the locus of a point which moves such that the sum of the squares of the distance from (a, 0) and (−a, 0) is 2c².

32) Find the equation of the line whose x-intercept is 'a' and y-intercept is b.

33) If the line 2x − 5y + 1 = 0 is perpendicular to (p + 1)x + (2p + 3)y + 3 = 0, find p.

34) Find the equation of the line passing through the point of intersection of 2x + 3y − 1 = 0 and 3x + 4y − 6 = 0 and parallel to the line 5x − y = 0.
I Semester B.C.A. Degree Examination, November/December 2014  
(CBCS) (Y2K14 Scheme) (Fresh) (2014-15 and Onwards)  
COMPUTER SCIENCE  
BCA 105T : Discrete Mathematics  

Time : 3 Hours  
Max. Marks : 100  

Instruction: Answer all Sections.  

SECTION – A  

I. Answer any ten of the following:  

(10x2=20)  

1) Define a power set. Illustrate with an example.  
2) If P = {1, 2} form the P x P x P.  
3) Define equivalence relation.  
4) Define Scalar Matrix with example.  
5) If A = \[
\begin{pmatrix}
2 & 1 \\
4 & -2
\end{pmatrix}
\]  
B = \[
\begin{pmatrix}
4 & 3 \\
2 & -1
\end{pmatrix}
\] find AB.  
6) Prove that 3 log 2 + log 5 = log 40.  
7) Define permutation.  
8) Define Coplanar vectors.  
9) Define slope of a line.  
10) Find the equation of the straight line passing through (2, 5) and having slope 4.  
11) Find the coordinates of the mid point which divides the join of (4, 3) and (−2, 7).  
12) Define order of a group.  

SECTION – B  

II. Answer any six of the following:  

(6x5=30)  

13) Verify whether (p → q) ↔ (¬q → ¬p) is a tautology.  
14) Prove that ¬(p ↔ q) = [¬(p → q) ∧ (q → p)].  
15) Consider f : R → R given by f(x) = 4x + 3. Show that f is invertible.  

P.T.O.
16) Verify Cayley Hamilton theorem for the matrix \( A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \).

17) Solve using Cramer’s rule
\[
\begin{align*}
3x + y + z &= 3 \\
2x + 2y + 5z &= -1 \\
x - 3y - 4z &= 2
\end{align*}
\]

18) Solve the equations \( 2x + 5y = 1 \), \( 3x + 2y = 7 \) using matrix method.

19) Find the eigen values and eigen vectors of \( A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \).

20) Let \( A = \mathbb{Z}^+ \), the set of positive integers. \( R = \{(a, b) \mid a \leq b\} \). Is \( R \) an equivalence relation.

SECTION – C

III. Answer any six of the following : \((6\times5=30)\)

21) If \( \log x - 2 \log \frac{6}{7} = \frac{1}{2} \log \frac{81}{16} - \log \frac{27}{196} \), find \( x \).

22) a) Find the number of different signals that can be generated by arranging atleast 3 flags in order (one below the other) on a vertical staff, if 6 different flags are available.

b) If \( \frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!} \), find \( x \).

23) a) Find \( r \) if \( 10^9 \text{P}_r = 2^9 \text{P}_r \).

b) In how many ways can the letters of the word ASSASSINATION be arranged so that all the S’s are together.

24) A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consist of (i) exactly 3 girls (ii) atleast 3 girls (iii) atmost 3 girls.

25) Prove that \( G = \{1, 5, 7, 11\} \) is a group under multiplication modulo 12.

26) If \( \vec{a} = 2\hat{i} + 3\hat{k} \) and \( \vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k} \), find \( \vec{a} \times \vec{b} \). Verify that \( \vec{a} \) and \( (\vec{a} \times \vec{b}) \) are perpendicular to each other.

27) Prove that \( \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0 \).

28) Using vector method show that the points \( A (2, -1, 3) \), \( B (4, 3, 1) \) and \( C (3, 1, 2) \) are collinear.
IV. Answer any four of the following: \(4 \times 5 = 20\)

29) Prove that the points \((4, -4), (8, 2), (14, -2)\) and \((10, -8)\) are the vertices of a square.

30) Find the equation of the locus of the point which moves such that its distance from \((0, 3)\) is twice its distance from \((0, -3)\).

31) Show that the line joining the points \((2, -3)\) and \((-5, 1)\) is
   a) Parallel to the line joining \((7, -1)\) and \((0, 3)\)
   b) Perpendicular to the line joining \((4, 5)\) and \((0, -2)\).

32) Find the equation of the straight line which passes through the point of intersection of the lines \(3x + y - 10 = 0\) and \(x + 7y - 10 = 0\) and parallel to the line \(4x - 3y + 1 = 0\).

33) Find the equations of the straight lines passing through the point \((4, -2)\) and making an angle of \(\frac{\pi}{4}\) with the line \(8x + 7y - 1 = 0\).

34) Prove that points \((2, 2)\) and \((-3, 3)\) are equidistant from the line \(x + 3y - 7 = 0\) and are on either side of the line.