I Semester B.C.A. Degree Examination, November/December 2018
(F+R) (CBCS) (2014-15 and Onwards)
COMPUTER SCIENCE
BCA 105 T : Discrete Mathematics

Time : 3 Hours  Max. Marks : 100

Instruction : Answer all Sections.

SECTION - A

I. Answer any ten of the following : 
(10×2=20)

1) If A = {c, d, e} and B = {a, b} find B × A.
2) Define an equivalence relation.
3) Define diagonal matrix with example.
4) Construct the truth table for ¬p → q.
5) If \[ A = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix} \] and \[ B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \] find \[ A + 3B. \]
6) Find the characteristic root of the matrix \[ A = \begin{bmatrix} 3 & 0 \\ 5 & 2 \end{bmatrix}. \]
7) If \[ \log_{2}^{6} = x, \] then find x.
8) If \[ ^{2}C_{9} = ^{2}C_{2}, \] find \[ ^{2}C_{2}. \]
9) Define abelian group.
10) If \[ \vec{a} = 2\hat{i} + \hat{j} - 3\hat{k} \] and \[ \vec{b} = 5\hat{i} + \hat{j} + 4\hat{k} \] find \[ \vec{a} + \vec{b}. \]
11) Find the distance between the points A(3, -1) and B(4, -2).
12) Find the equation of the line with slope 3 and cutting off an intercept 2 on y-axis.

SECTION - B

II. Answer any six of the following : 
(6×5=30)

13) If \[ A = \{1, 2, 3, 4\} \] B = \{3, 4, 5\} and C = \{3, 5, 6, 7\} then verify
\[ A \times (B \cup C) = \{A \times B\} \cup \{A \times C\}. \]
14) If \[ f : R \rightarrow R \] is defined \[ f(x) = 7x - 8 \] prove that f is invertible and find \[ f^{-1}. \]

P.T.O.
15) Prove that \((-q \rightarrow \neg p) \iff (p \rightarrow q)\) is a tautology.

16) Verify whether \((p \land \neg q) \land (\neg p \lor q)\) is a contradiction or not.

17) Prove that \([p \land (q \lor r)] \equiv [(p \land q) \lor (p \land r)]\).

18) Find the inverse of the matrix \(A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\).

19) Solve \(2x + 3y + z = 9\), \(4x + y = 7\) and \(x - 3y - 7z = 6\) using Cramer's rule.

20) State and verify the Cayley-Hamilton theorem for the matrix \(A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}\).

SECTION - C

III. Answer any six questions:

21) If \(x = \log_{2a}^{a}\), \(y = \log_{3a}^{a}\), \(z = \log_{4a}^{a}\) then prove that \(1 + xyz = 2yz\).

22) i) If \(\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}\) find \(x\).

ii) Find \(n\) if \(2np_s = np_s\).

23) Prove that \(G = \{2, 4, 6, 8\}\) is an abelian group under multiplication modulo 10.

24) Prove that \(H = \{1, -1\}\) is a subgroup of \(G = \{1, -1, i, -i\}\) under multiplication.

25) If \(\vec{a} = i + 2j - 3k\), \(\vec{b} = 2i - j - 4k\) find \((\vec{a} \cdot \vec{b})\) \((4\vec{a} + 3\vec{b})\).

26) Find the area of the triangle whose vertices are \(A (1, 3, 2)\), \(B (-1, 4, -1)\) and \((-2, 3, -5)\) using vector method.

27) If the vectors \(\vec{a} = 3i - 4j + mk\), \(3i + j - k\) and \(\vec{c} = 2i - 2j + 4k\) are coplanar find \(m\).

28) In how many different ways can the letters of word "MISSISSIPPI" be arranged. In how many of these arrangements do the four 'I's not come together?
IV. Answer any four of the following: (4x5=20)

29) Prove that the points (3, 4), (6, 8) (9, 8) and (6, 4) form a parallelogram.
30) The three vertices of a rhombus taken in order are (2, −1), (3, 4) (−2, 3).
    Find the fourth vertex.
31) Find the equation to the perpendicular bisector of the line joining the points
    (−1, 5) and (2, 4).
32) Derive the equation of the straight line whose x-intercept is ‘a’ and
    y-intercept is ‘b’.
33) Find the value of k if the lines
    i) $3x + 2y + 1 = 0$ and $kx + 2y − 1 = 0$ are parallel
    ii) $5x − 4y + 8 = 0$ and $4x + ky + 3 = 0$ are perpendicular.
34) Find the equation of the straight line which is passing through the intersection
    of the lines $2x − 3y − 4 = 0$ and $2x + 2y − 1 = 0$ and perpendicular to the line
    $x + 4y − 8 = 0$. 
I Semester B.C.A. Degree Examination, Nov./Dec. 2017  
(2014-15 and Onwards) (F + R) (CBCS)  
BCA – 105 T : DISCRETE MATHEMATICS

Time : 3 Hours  
Max. Marks : 100

Instruction: Answer all Sections.

SECTION – A

1. Answer any ten of the following: (10x2=20)
   1) If A = \{2, 3, 4, 5\} and B = \{0, 1, 2, 3\}, find A \cap B.
   2) If A = \{x^2 - 5x + 6 = 0, x \in N\} and B = \{3, 4, 5\}, find A \times B.
   3) Define contradiction.
   4) Define unit matrix with example.
   5) If \( A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \) and \( B = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix} \), find 2A + 3B.
   6) Find the characteristic roots of the matrix \( A = \begin{bmatrix} 3 & 0 \\ 2 & 5 \end{bmatrix} \).
   7) Prove that \( \log_{3a} 2a \cdot \log_{4a} 3a = \frac{1}{2} \).
   8) If \( ^nC_{30} = ^nC_5 \), find 'n'.
   9) Define group.
   10) If \( \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k} \), find \( |2\vec{a} + \vec{b}| \).
   11) Find the distance between the points A(2, -3) and B(4, 5).
   12) Write the slope of the line 4x - 3y + 2 = 0.
SECTION - B

II. Answer any six of the following: (6x5=30)

13) In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

14) If \( f : \mathbb{R} \to \mathbb{R} \) is defined by \( f(x) = 4x + 5 \) prove that \( f \) is one-one and onto.

15) Prove that \( (p \to q) \leftrightarrow (\sim q \to \sim p) \) is a tautology.

16) Prove that \( p \land (q \lor r) \equiv (p \land q) \lor (p \land r) \).

17) Write the converse, inverse and contrapositive of "If two triangles are congruent, then they are similar".

18) If \[ A = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad B = [2 \ 3 \ 5] \] prove that \((AB)^T = B^T A^T\).

19) If \[ A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} \], then find \( A^{-1} \) using Cayley-Hamilton theorem.

20) Solve \( 5x + 2y = 4 \), \( 7x + 3y = 5 \) using Cramer's rule.

SECTION - C

III. Answer any six of the following: (6x5=30)

21) If \( \log \left( \frac{a+b}{3} \right) = \frac{1}{2} \left( \log a + \log b \right) \), then prove that \( a^2 + b^2 = 7ab \).

22) Prove that the set \( G = \{1, -1, i, -i\} \) is a group under multiplication.

23) Prove that \( H = \{0, 2, 4\} \) is a subgroup of \( G = \{0, 1, 2, 3, 4, 5\} \) under addition modulo 6.

24) How many different words can be formed with the letters of the word "MISSISSIPPI"? In how many of these four I's do not come together?

25) If \( ^nC_3 : ^nC_2 = 44 : 3 \) find \( n \).

26) If \( \vec{a} = 2\hat{i} + \hat{j} + 4\hat{k} \), \( \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k} \) and \( \vec{c} = 3\hat{i} + \hat{j} + 4\hat{k} \) find \( \vec{a} \cdot (\vec{b} \times \vec{c}) \).

27) Find the area of the triangle whose vertices are \( A(3, 2, 1) \), \( B(4, -1, 2) \) and \( C(-1, 3, 2) \) using vector method.

28) Find the value of \( m \) if \( \vec{a} = m\hat{i} - 3\hat{j} + 4\hat{k} \), \( \vec{b} = \hat{i} + 3\hat{j} + \hat{k} \) and \( \vec{c} = 2\hat{i} + \hat{j} + \hat{k} \) are coplanar.
SECTION – D

IV. Answer any four of the following: (4x5=20)

29) Prove that the points A(3, -4), B(4, 2), C(5, -4) and D(4, -10) form vertices of a rhombus.

30) If a vertex of triangle is (1, 1) and the mid-point points of two sides through this vertex are (-1, 2) and (3, 2) then find the centroid of the triangle.

31) Find the acute angle between the lines 2x - y + 13 = 0 and 2x - 6y + 7 = 0.

32) The angle between two lines is $\frac{\pi}{4}$ and the slope of one line is $\frac{1}{2}$. Find the slope of the other line.

33) Find the point of intersection of the straight lines 3x - 4y - 1 = 0 and 5x - 7y - 1 = 0.

34) Prove that the point (-1, 3) is equidistant from the lines x + y - 3 = 0 and 7x - y + 5 = 0.
I Semester B.C.A. Degree Examination, November/December 2016  
(CBCS) (F+R)  
(2014-15 & Onwards)  
BCA – 105 : DISCRETE MATHEMATICS

Time : 3 Hours  Max. Marks : 100

Instruction : Answer all Sections.

SECTION – A

1. Answer any ten : (10x2=20)

1) If A = \{ x | x \in N and x<3 \} and B = \{ 0, 1, 3 \}. Find A \cap B.

2) If A = \{ 1, 2, 3 \}, B = \{ 3, 4, 5 \} and C = \{ 0, 2, 3 \}, find (A \cap B) \times C.

3) Construct truth table for the proposition p \lor \neg q.

4) Find x, y, z if
\[
\begin{bmatrix}
4 - y & 3 \\
x & 5
\end{bmatrix} =
\begin{bmatrix}
-1 & z + 1 \\
1 & 5
\end{bmatrix}.
\]

5) If \(A = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}\) and \(B = \begin{bmatrix} 2 & 0 & 3 \\ 3 & 1 & 4 \end{bmatrix}\), find \(AB\).

6) Find the characteristic equation of the matrix \(\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}\).

7) Prove that \(\log_a a \cdot \log_b b \cdot \log_c c = 1\).

8) Find n if \((nP_3) = nP_5\).

9) On the set of integers \(Z\), the binary operation \(*\) is defined by
\[
a * b = \frac{ab}{3}, \quad \forall a, b \in Z.
\]
Find identity element.

10) If \(\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}\), \(\vec{b} = \hat{i} - \hat{j} + 2\hat{k}\) find unit vector along \(\vec{a} - \vec{b}\).

P.T.O.
11) Find the midpoint of line joining (−2, 8) and (1, −2).

12) Find the equation of the line passing through (−1, 2) and having slope 3.

II. Answer any six of the following:

13) If \{A = (1, 4), B = (2, 3, 6), C = (2, 3, 7)} then verify that \( A \times (B - C) = (A \times B) - (A \times C). \)

14) Show that the function \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = 4x + 3 \) is invertible. Find the inverse of \( f. \)

15) Show that \( p \lor (q \land r) \leftrightarrow [(p \lor q) \land (p \lor r)] \) is a tautology.

16) If \( (p \to q) \land (p \land r) \) is given to be false, find the truth values of \( p, q, r. \)

17) Write the truth table of \( (p \lor q) \lor \lnot p. \) Show that the compound propositions \( p \land q \) and \( \lnot (p \to \lnot q) \) are logically equivalent.

18) Find the inverse of the matrix \( A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & 3 & 4 \\ 0 & -1 & 1 \end{bmatrix}. \)

19) Using Cramer's rule solve \( 3x - y + 2z = 13; \quad 2x + y - z = 3; \quad x + 3y - 5z = -8. \)

20) Verify Cayley Hamilton theorem for the matrix \( \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}. \)

III. Answer any six of the following:

21) If \( \log_{\frac{a-b}{5}} = \frac{1}{2} \log a + \log b, \) show that \( a^2 + b^2 = 27 \) \( ab. \)

22) Find the number of three digit even numbers that can be formed using 2, 3, 4, 5, 6 repetitions being not allowed.

23) If \( \binom{n+2}{6} : \binom{n-2}{4} = 57 : 16 \) find \( n. \)
24) Prove that the set $G = \{ 3n \mid n \in \mathbb{Z} \}$ is an abelian group w.r.t. addition.

25) Prove that the set $G = \{2, 4, 6, 8\}$ is an abelian group w.r.t. multiplication modulo 10.

26) If $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$, find $(\vec{a} + 2\vec{b}) \cdot (2\vec{a} - \vec{b})$.

27) Show that the points $A(1,2,3)$, $B(2,3,1)$ and $C(3,1,2)$ are vertices of an equilateral triangle.

28) If the vectors $4\hat{i} + 4\hat{j} + m\hat{k}$, $7\hat{i} + 2\hat{j} + n\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then find 'm'.

**SECTION - D**

IV. **Answer any four** of the following. $(4 \times 5 = 20)$

29) Prove that the points $(6, 4)$, $(7, -2)$, $(5, 1)$, $(4, 7)$ form vertices of a parallelogram.

30) The three vertices of a parallelogram taken in order are $(8,5)$, $(-7, -5)$ and $(-5, 5)$. Find the co-ordinate of the fourth vertex.

31) Find the equation of the locus of a point which moves such that its distance from X-axis is twice its distance from Y-axis.

32) Derive the equation of the straight line whose x-intercept is 'a' and y-intercept is 'b'.

33) Find 'K' for which the lines $2x - ky + 1 = 0$ and $x + (k+1)y - 1 = 0$ are perpendicular.

34) Find the equation of straight line which is passing through intersection of the lines $2x - 3y - 4 = 0$ and $2x + 2y - 1 = 0$ and perpendicular to the line $x + 4y - 8 = 0$. 


I. Answer any ten : (10×2=20)

1) If \( A = \{2, 3, 4, 8\}, B = \{1, 3, 4\} \) and \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \).
Verify \( A \cap B = A \cap B \).

2) If \( A = \{2, 3, 5\}, B = \{4, 5, 6\} \) and \( C = \{1, 2\} \) find \( A \times B \).

3) Define Tautology.

4) Define diagonal matrix.

5) If \( 2Y + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} \), find \( Y \).

6) State Cayley Hamilton theorem.

7) If \( \log_{7} x + \log_{7} x^2 + \log_{7} x^3 = 6 \), find \( x \).

8) Define combination.

9) Define Abelian group.

10) If \( \vec{a} = 2i + 3j - 4k, \vec{b} = 3i - 4j - 5k \) find \( |\vec{a} + \vec{b}| \).

11) Find the distance between the point, \( A = (-7, 4) \) and \( B = (-5, -1) \).

12) Find the equation of the line with slope 2 and cutting off an intercept 3 on Y-axis.
SECTION - B

II. Answer any six of the following: \(6 \times 5 = 30\)

13) If \(A = \{a, b, c, d\}, B = \{c, d\}\) and \(C = \{d, e\}\) find \(A - B, (A - B) \cap (B - C), B \times C.\)

14) If \(f : \mathbb{R} \rightarrow \mathbb{R}\) is defined by \(f(x) = 2x + 5\), prove that \(f\) is one-one and onto.

15) Prove that \((p \land q) \land (p \lor q)\) is a contradiction.

16) Write the converse, inverse and contra positive of

   "If I work hard then I get a grade".

17) Find the truth values of the propositions \(p, q\) and \(r\), if the compound proposition \((p \rightarrow \neg q) \rightarrow r\) is false.

18) If \(2A + B = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}, A - 2B = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}\) then find \(A\) and \(B.\)

19) If \(A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}\), find \(A^{-2}\) using Cayley Hamilton theorem.

20) Solve the equations \(5x + 2y = 4, 7x + 3y = 5\) using Matrix method.

SECTION - C

III. Answer any six of the following: \(6 \times 5 = 30\)

21) If \(\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)\), show that \(a = b.\)

22) In how many ways the letters of the word "EVALUATE" be arranged so that all vowels are together.

23) If \(^{15}C_{r+3} = ^{15}C_{2r-3}\), find \(r.\)

24) If \(G = \{3^n : n \in \mathbb{Z}\}\), prove that \(G\) is an abelian group under multiplication.

25) Prove that \(G = \{1, 5, 7, 11\}\) is a group under multipication modulo 12.

26) Find the value of \(\lambda\) for which the vectors \(\vec{a} = 3i + j - 2k\) and \(\vec{b} = i + \lambda j - 3k\)

   are perpendicular to each other.

27) Find the area of the triangle whose vertices are \(A(1, 2, 3), B(2, 5, 1)\) and \(C(-1, 1, 2)\) using vector method.

28) If the vectors \(2i - 3j + mk, 2i + j - k\) and \(6i - j + 2k\) are coplanar, find \(m.\)
SECTION – D

IV. Answer any four of the following: 

(4x5=20)

29) Show that the points (3, 2), (0, 5), (−3, 2) and (0, −1) are the vertices of a square.

30) Find the ratio in which the x-axis divides the line segment joining the points (7, −3) and (5, 2).

31) Find the equation of the locus of a point which moves such that the sum of the squares of the distance from (a, 0) and (−a, 0) is 2c^2.

32) Find the equation of the line whose x-intercept is ‘a’ and y-intercept is b.

33) If the line 2x − 5y + 1 = 0 is perpendicular to (p + 1)x + (2p + 3)y + 3 = 0, find p.

34) Find the equation of the line passing through the point of intersection of 2x + 3y − 1 = 0 and 3x + 4y − 6 = 0 and parallel to the line 5x − y = 0.
I Semester B.C.A. Degree Examination, November/December 2014  
(CBCS) (Y2K14 Scheme) (Fresh) (2014-15 and Onwards)  
COMPUTER SCIENCE  
BCA 105T : Discrete Mathematics

Time : 3 Hours  
Max. Marks : 100

**Instruction**: Answer all Sections.

SECTION – A

I. Answer any ten of the following :  

1) Define a power set. Illustrate with an example.
2) If P = {1, 2} form the P x P x P.
3) Define equivalence relation.
4) Define Scalar Matrix with example.

5) If $A = \begin{pmatrix} 2 & 1 \\ 4 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 3 \\ 2 & -1 \end{pmatrix}$ find AB.
6) Prove that $3 \log 2 + \log 5 = \log 40$.
7) Define permutation.
8) Define Coplanar vectors.
9) Define slope of a line.

10) Find the equation of the straight line passing through (2, 5) and having slope 4.
11) Find the coordinates of the mid point which divides the join of (4, 3) and (−2, 7).
12) Define order of a group.

SECTION – B

II. Answer any six of the following :  

13) Verify whether $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ is a tautology.
14) Prove that $\neg(p \leftrightarrow q) = \neg[(p \rightarrow q) \land (q \rightarrow p)]$.
15) Consider $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 4x + 3$. Show that f is invertible.

P.T.O.
16) Verify Cayley Hamilton theorem for the matrix \( A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \).

17) Solve using Cramer's rule
\[
\begin{align*}
3x + y + z &= 3 \\
2x + 2y + 5z &= -1 \\
x - 3y - 4z &= 2
\end{align*}
\]

18) Solve the equations \( 2x + 5y = 1, 
3x + 2y = 7 \) using matrix method.

19) Find the eigen values and eigen vectors of \( A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \).

20) Let \( A = Z^* \), the set of positive integers. \( R = \{(a, b) \mid a \leq b \} \). Is \( R \) an equivalence relation.

**SECTION – C**

III. Answer any six of the following: (6x5=30)

21) If \( \log x - 2\log \frac{6}{7} = \frac{1}{2} \log \frac{81}{16} - \log \frac{27}{196} \) find \( x \).

22) a) Find the number of different signals that can be generated by arranging at least 3 flags in order (one below the other) on a vertical staff, if 6 different flags are available.

b) If \( \frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!} \) find \( x \).

23) a) Find \( r \) if \( 10P_r = 2^9 P_r \).

b) In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together.

24) A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consist of (i) exactly 3 girls (ii) atleast 3 girls (iii) atmost 3 girls.

25) Prove that \( G = \{1, 5, 7, 11\} \) is a group under multiplication modulo 12.

26) If \( \vec{a} = \hat{i} - 2\hat{j} + 3\hat{k} \) and \( \vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k} \) find \( \vec{a} \times \vec{b} \). Verify that \( \vec{a} \) and \( (\vec{a} \times \vec{b}) \) are perpendicular to each other.

27) Prove that \( \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0. \)

28) Using vector method show that the points \( A (2, -1, 3), B (4, 3, 1) \) and \( C (3, 1, 2) \) are collinear.
IV. Answer any four of the following:

29) Prove that the points (4, −4), (8, 2), (14, −2) and (10, −8) are the vertices of a square.

30) Find the equation of the locus of the point which moves such that its distance from (0, 3) is twice its distance from (0, −3).

31) Show that the line joining the points (2, −3) and (−5, 1) is
   a) Parallel to the line joining (7, −1) and (0, 3)
   b) Perpendicular to the line joining (4, 5) and (0, −2).

32) Find the equation of the straight line which passes through the point of intersection of the lines 3x + y − 10 = 0 and x + 7y − 10 = 0 and parallel to the line 4x − 3y + 1 = 0.

33) Find the equations of the straight lines passing through the point (4, −2) and making an angle of $\frac{\pi}{4}$ with the line 8x + 7y − 1 = 0.

34) Prove that points (2, 2) and (−3, 3) are equidistant from the line $x + 3y − 7 = 0$ and are on either side of the line.